

Two-photon exchange amplitudes for the elastic ep scattering at $Q^2 = 2.5 \text{ GeV}^2$ from the experimental data

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We extract two-photon exchange amplitudes for the elastic electron-proton scattering at $Q^2 = 2.5 \text{ GeV}^2$ from the unpolarized cross-section and recent polarization transfer measurements. There are three independent amplitudes, but only one of them, $\delta\mathcal{G}_M$, can be determined with a reasonable accuracy (about 10%). The result is in good agreement with theoretical predictions. Rough estimates for two other amplitudes are obtained.

Introduction. In last years, a lot of experimental and theoretical effort was made to study two-photon exchange (TPE) in the elastic electron-proton scattering. This activity was motivated by the discovery of the problem in the proton form factor measurements: values of form factor ratio G_E/G_M obtained by Rosenbluth separation and polarization transfer methods were in strong disagreement. It is now widely accepted that the discrepancy is caused by TPE, but still no direct experimental observations of TPE exist.

Previously, several attempts were made to extract values of TPE amplitudes from available experimental data in more or less model-independent way [1, 2]. However, lack of precise data and/or theoretical understanding of TPE prevented from obtaining sufficiently accurate estimates for the amplitudes. In particular, it was difficult to determine the dependence of TPE amplitudes on the kinematical parameter ε , since the ε dependence of polarization observables was not known experimentally.

Recently, a search for TPE effects in polarization observables was reported [3]. In this experiment, ratio of transverse and longitudinal proton polarization components (polarization ratio) was measured with significantly improved precision for $Q^2 = 2.5 \text{ GeV}^2$ and wide range of the parameter ε .

In the present paper we use latest experimental data to determine TPE amplitudes at $Q^2 = 2.5 \text{ GeV}^2$ following the ideas of Ref. [2] with some improvements (described below). We will try to obtain as much information on TPE amplitudes as possible, while avoiding unnecessary assumptions. In our analysis we only assume that TPE is the sole reason for the discrepancy between cross-section and polarization data, and rely on the following experimental and theoretical facts:

1. The reduced cross-section exhibits no or small non-linearity in ε ([4, 5], also verified in the present work).
2. The polarization ratio does not vary significantly with ε [3].
3. TPE amplitudes must vanish at $\varepsilon \rightarrow 1$ (because they can be represented by convergent dispersion integral; $\varepsilon \rightarrow 1$ implies $s \rightarrow \infty$, where s is c.m. energy squared).

The latter point was missing in Ref. [2].

Analysis of cross-section data. There were no recent cross-section measurements in the Q^2 region of our interest, so we have to use older data. These data were analyzed before, but for our paper to be self-contained we repeat such an analysis here. We have selected data in the range $2.2 \text{ GeV}^2 < Q^2 < 2.8 \text{ GeV}^2$ [6]. The corresponding reduced cross-sections were first multiplied by $(1 + Q^2/0.71 \text{ GeV}^2)^4$, to eliminate most of the Q^2 dependence. Then, since in Born approximation the reduced cross-section is

$$\sigma_R = \tau G_M^2 + \varepsilon G_E^2 \quad (1)$$

where G_E and G_M are electric and magnetic form factors, $\tau = Q^2/4M^2$ and M is proton mass, the resulting values were fitted with the function

$$A + B\varepsilon + C(Q^2 - 2.5 \text{ GeV}^2) \quad (2)$$

The last term takes into account Q^2 dependence of τG_M^2 term in σ_R . We obtain rather acceptable fit with $\chi^2 = 39$ for 28 d.o.f., indicating that the linearity of σ_R in ε is indeed supported by the data. We will mainly need the quantity

$$R_{LT}^2 = \tau B/A \quad (3)$$

which would be equal to $(G_E/G_M)^2$ at $Q^2 = 2.5 \text{ GeV}^2$ in Born approximation. We obtain $R_{LT}^2 = 0.1020 \pm 0.0057$, in agreement with the results Ref. [5] (0.1015). In further calculations we use the first value.

Extraction of TPE amplitudes. We will use mostly the same notation as in Ref. [2]. We denote particle momenta according to

$$e(k) + p(p) \rightarrow e(k') + p(p'), \quad (4)$$

and define $q = p' - p$, $P = (p + p')/2$, $K = (k + k')/2$, $Q^2 = -q^2$. In presence of TPE, elastic electron-proton scattering amplitude has the form

$$\mathcal{M} = \frac{4\pi\alpha}{Q^2} \bar{u}' \gamma_\mu u \bar{U}' \left(\tilde{F}_1 \gamma^\mu - \tilde{F}_2 [\gamma^\mu, \gamma^\nu] \frac{q_\nu}{4M} + \tilde{F}_3 K_\nu \gamma^\nu \frac{P^\mu}{M^2} \right) U \quad (5)$$

where α is fine structure constant, u, u' (U, U') are initial and final electron (proton) spinors, and \tilde{F}_i are scalar

invariant amplitudes. It is convenient to introduce linear combinations [2]

$$\begin{aligned}\mathcal{G}_E &= \tilde{F}_1 - \tau\tilde{F}_2 + \nu\tilde{F}_3/4M^2 = G_E + \delta\mathcal{G}_E \\ \mathcal{G}_M &= \tilde{F}_1 + \tilde{F}_2 + \varepsilon\nu\tilde{F}_3/4M^2 = G_M + \delta\mathcal{G}_M \\ \mathcal{G}_3 &= \nu\tilde{F}_3/4M^2 = \delta\mathcal{G}_3\end{aligned}\quad (6)$$

where $\nu = 4PK$ and prefix δ indicates TPE contribution. The TPE amplitudes $\delta\mathcal{G}_i$ are complex, but only their real parts contribute to the observables discussed here. Everywhere below, speaking of the amplitudes, we will mean their real parts. Neglecting terms of order α^2 , the reduced cross-section and polarization ratio can be written as

$$\sigma_R = G_M^2 \left\{ \tau + \varepsilon R_0^2 + 2\tau \frac{\delta\mathcal{G}_M}{G_M} + 2\varepsilon R_0^2 \frac{\delta\mathcal{G}_E}{G_E} \right\} \quad (7)$$

$$R = R_0 \left\{ 1 + \frac{\delta\mathcal{G}_E}{G_E} - \frac{\delta\mathcal{G}_M}{G_M} - \frac{\varepsilon(1-\varepsilon)}{1+\varepsilon} \frac{\delta\mathcal{G}_3}{G_M} \right\} \quad (8)$$

where $R_0 = G_E/G_M$. Note that our definition of R does not include a factor of $\mu \approx 2.793$, thus R (and R_0) is rather small quantity (≈ 0.25 for $Q^2 = 2.5$ GeV²). Utilizing this fact we will neglect last term in Eq.(7) (the validity of this approximation will be checked afterwards). Then, as it was argued in Ref. [2], the observed cross-section linearity in ε forces us to parameterize TPE amplitude $\delta\mathcal{G}_M$ as a linear function of ε . To vanish at $\varepsilon \rightarrow 1$, it must have the form

$$\delta\mathcal{G}_M/G_M = a(1-\varepsilon) \quad (9)$$

Then we have

$$\sigma_R = G_M^2 \{ \tau + \varepsilon R_0^2 + 2\tau a(1-\varepsilon) \} \quad (10)$$

and the cross-section slope is

$$R_{LT}^2 = \frac{R_0^2 - 2\tau a}{\tau(1+2a)} \quad (11)$$

from which we obtain

$$a = \frac{R_0^2 - R_{LT}^2}{2(\tau + R_{LT}^2)} \quad (12)$$

Together with Eq.(9), this fully determines the amplitude $\delta\mathcal{G}_M$. As a first approximation, we replace R_0^2 by experimental value of polarization ratio, $R = 0.6923 \pm 0.0058$ [3], and obtain numerically

$$a = -0.0250 \pm 0.0035 \quad (13)$$

Thus extracted amplitude $\delta\mathcal{G}_M/G_M$ is shown in Fig. 1 with 1σ error band. The theoretical prediction [7, 8] is also shown and agrees rather well with our result.

Now we will take a closer look on the polarization ratio R , which allows us to get some information about the amplitude $\delta\mathcal{G}_E$. First, we note that the last term in Eq.(8) should be very small, because the factor $\frac{\varepsilon(1-\varepsilon)}{1+\varepsilon}$ is

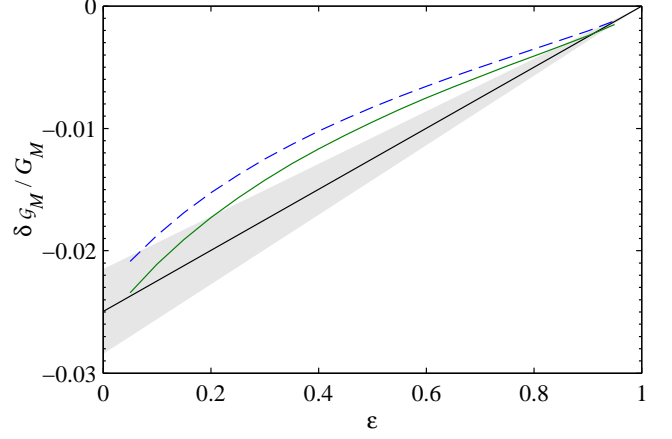


FIG. 1: Extracted TPE amplitude $\delta\mathcal{G}_M/G_M$ (grey, 1σ band) and its theoretical estimates, calculated with: elastic intermediate state only [7] (dashed), elastic + Δ resonance [8] (solid).

not greater than 0.18 for $0 < \varepsilon < 1$. Thus we are left with

$$R = R_0 \left\{ 1 + \frac{\delta\mathcal{G}_E}{G_E} - \frac{\delta\mathcal{G}_M}{G_M} \right\} \quad (14)$$

Since both $\delta\mathcal{G}_M$ and $\delta\mathcal{G}_E$ vanish at $\varepsilon \rightarrow 1$, we obviously have

$$R_0 = R|_{\varepsilon=1}, \quad \delta\mathcal{G}_E - R_0\delta\mathcal{G}_M = R - R_0 \quad (15)$$

The experiment says that, at $Q^2 = 2.5$ GeV², there is no significant variation of R with ε [3]. This implies

$$R \approx R_0 \quad \text{and} \quad \delta\mathcal{G}_E \approx R_0\delta\mathcal{G}_M \quad (16)$$

(which also justifies using R instead of R_0 in Eq.(12)). Now we can cross-check that the amplitude $\delta\mathcal{G}_E$ has small impact on the cross-section. Using Eqs.(9,12,16), we calculate the corresponding correction (the last term of Eq.(7)) and subtract it from the data. Then we repeat cross-section fitting and extraction of $\delta\mathcal{G}_M$, as described above. We obtain $a = -0.0248$, i.e. practically no change with respect to (13).

Eqs.(16) are, of course, a rough estimate. In particular, they do not allow to estimate the uncertainty of $\delta\mathcal{G}_E$. An accurate \mathcal{G}_E extraction with estimation of uncertainties requires determination of the small quantity $\delta R = R - R_0$, for which the present data hardly suffice. Nevertheless, note that theoretical calculations also show relative smallness of δR , which arises from significant cancellation between proton and Δ resonance contributions (Fig. 2).

In the experiment [3], one more quantity was measured: longitudinal polarization of the final proton, P_l . In principle, this data could help to determine the remaining amplitude $\delta\mathcal{G}_3$.

The TPE correction to P_l is given by

$$\delta P_l = -2\varepsilon P_l \left\{ \frac{R_0^2 \delta R}{\varepsilon R_0^2 + \tau} + \frac{\varepsilon}{1+\varepsilon} \frac{\delta\mathcal{G}_3}{G_M} \right\} \quad (17)$$

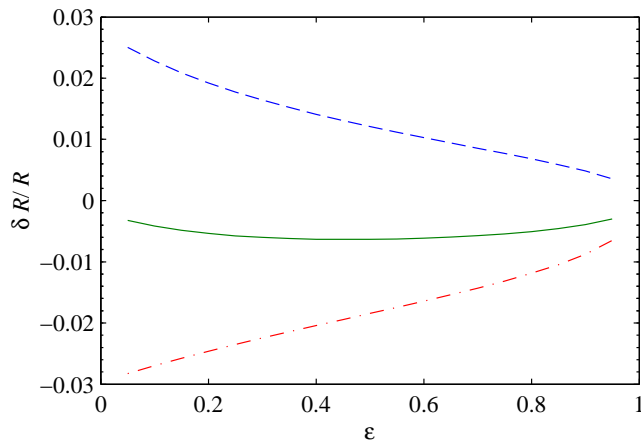


FIG. 2: TPE correction to polarization ratio $\delta R/R$, contribution of proton (dashed), Δ resonance (dash-dotted), and total (solid).

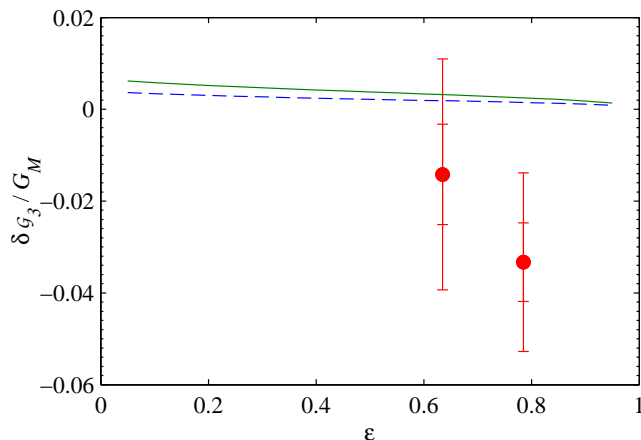


FIG. 3: Extracted amplitude $\delta \mathcal{G}_3/G_M$. Statistical and systematic errors are added in quadrature, inner bars — pure statistical errors. Theoretical calculations with: elastic intermediate state only [7] (dashed), elastic + Δ resonance [8] (solid).

where $\delta R = R - R_0$ is TPE correction to polarization ratio according to Eq.(14). As $R_0^2 \ll 1$, the first term is negligible, and the deviation of P_L from its Born value is governed by the amplitude $\delta \mathcal{G}_3$. However, two available data points are too few to make any statements about ε dependence of $\delta \mathcal{G}_3$. We can only compute $\delta \mathcal{G}_3$ at the ε values of experimental data. The results are shown in Fig. 3. Obviously, their precision is insufficient to make a meaningful comparison with theory. The obtained values are compatible with zero, and do not contradict theoretical estimates as well.

Conclusions. In summary, we have tried to extract TPE amplitudes for the elastic electron-proton scattering at $Q^2 = 2.5 \text{ GeV}^2$ solely from the experimental data on cross-sections and polarization observables. Having defined three independent amplitudes $\delta \mathcal{G}_M$, $\delta \mathcal{G}_E$ and $\delta \mathcal{G}_3$, we found that the effect of these amplitudes on the observables is “decoupled”: the cross-section is mainly influenced by $\delta \mathcal{G}_M$, the polarization ratio — by $\delta \mathcal{G}_E$ and $\delta \mathcal{G}_M$, longitudinal polarization component — by $\delta \mathcal{G}_3$. The amplitude $\delta \mathcal{G}_M$ can be extracted with approximately 10% accuracy and is in good agreement with theoretical calculations. The weakness of polarization ratio variation with ε implies approximate equality $\delta \mathcal{G}_E/G_E \approx \delta \mathcal{G}_M/G_M$. As to the amplitude $\delta \mathcal{G}_3$, present experimental data are consistent with $\delta \mathcal{G}_3 = 0$. To allow for more accurate extraction of $\delta \mathcal{G}_E$ and $\delta \mathcal{G}_3$, further polarization measurements at different ε are clearly needed. It is also worth noting that momentum transfer value $Q^2 = 2.5 \text{ GeV}^2$ was not good choice for the experiment [3]: just in this region TPE correction to polarization ratio is especially small because elastic and Δ resonance contributions almost cancel each other.

Recently, a preprint [9] appeared, in which the same problem was considered. However, authors have used certain parameterization of polarization component P_L , which, to our opinion, is not well motivated by experimental data. Their results strongly differ from ours as well as from theoretical calculations.

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